RESTRICTED

In the PRO

TD8

DEPARTMENT OF THE SCIENTIFIC ADVISER TO THE ARMY COUNCIL

MILITARY OPERATIONAL RESEARCH REPORT

REPORT No.33.

LIBRARY
Date rec'd.
Accession No. Nil

TANK BATTLE ANALYSIS

NOVEMBER 1946

TANK BATTLE ANALYSIS.

PREPARED BY - Major G.Gee.
Major J.M.Murdoch Mitchesen.

A B S T R A C T .

This report is a mathematical analysis of some two hundred small-scale tank v tank and tank v anti-tank gun actions of the late war. The object is to show quantitatively how the chance of success in battle for these weapons depended upon certain primary factors. The particular factors taken are those susceptible to such analysis and are:-

(a) Type of tank (or gun).
(b) Number of tanks (or guns).
(c) Range of engagement.
(d) Effect of firing first.

From the analysis of the effect of factor (a) "Efficiency Factors" for certain equipments have been calculated. These are purely relative and express the superiority of one type of tank (or gun) over another. The calculation of the Efficiency Factors is based on a comparison of the percentage casualties inflicted by the weapons concerned. It follows that they are an index of the efficiency of the combination tank plus crew (or gun plus crew) in battle. No attempt is made, indeed it cannot be made on the evidence considered here, to decide how this index depends upon the diverse detailed factors which affect tank battles. Tank design, armament, commander's and crew's morale, tactical handling, terrain, conditions of light - all such as these are averaged out and independent conclusions concerning any one of them cannot be drawn from the figures.

The types of weapons involved in the actions considered are as follows:-

(a) Tanks.
Allied - Sherman 75 mm. Cromwell 75 mm.
Sherman 17 pdr. Challenger 17 pdr.
German - PZKW Mk.III, Mk IV, Mk.V and Mk.VI.
(b) Anti-tank Guns.
Allied S.P. - 75 mm.
Allied towed guns - 17 pdr. and 6 pdr.
German S.P. - 75 mm. and 88 mm.
German towed guns - 75 mm. and 88 mm.

Unfortunately it was not possible to distinguish between Allied 17 pdr. and 6 pdr. towed guns nor between German 75 mm. and 88 mm. guns and guns of different calibre have had to be grouped together. The conclusion below should be read accordingly. Similarly the expression "German tanks"

includes tanks of all four marks unless otherwise stated. It may be noted that of the Allied [illegible] approximately 75% were 17 pdr. and 25% were 6 pdr; the German guns were roughly 50% 75 mm. and 50% 88 mm.

In this report analyses are made on groups of actions varying in size from 16 to 30. The size of the sample was limited by the fact that in War Diaries small scale actions were rarely described in sufficient detail for our purpose.

Since the only information available was from Allied sources it follows that

(a) the number of enemy successes recorded is small and lacking in detail.
(b) the figures given for numbers and types of enemy weapons and range at which used are only estimates which cannot be considered wholly reliable.

For these reasons the conclusions drawn must be regarded as tentative. However, the report is an illustration of a technique which might be applied to tank battle analysis and it shows what data is necessary for such an analysis to be made adequately.

The main conclusions are:-

(a) S.P. guns are more effective than towed A/Tk guns, the Efficiency Factors being 3 in the case of Allied weapons and 2 for German weapons.
(b) German Mark V and VI tanks were more effective than Mark III and IV tanks against Allied S.P. guns by a factor of 4.
(c) In tank versus tank engagements, for the chance of success to be equal for either side, Allied tanks would have to outnumber the German tanks by some 30% or about one-third.
(d) In an engagement between Allied tanks and German anti-tank guns the tanks would have to outnumber the guns by about two to one, for the chance of success to be equal for either side.
(e) The mean "Success Range" of the Allied 17 pdr. tank against German tanks was nearly twice that of the Allied 75 mm. tank (1100 yards as against 580 yards).
(f) In tank versus tank engagements German tanks were successful at an average range of 1290 yards whereas Allied tanks were successful at an average range of only 750 yards.
(g) Allied anti-tank guns, (both S.P. and towed guns), were successful against tanks at significantly greater ranges than were the corresponding German weapons (1090 yards against 330 yards for S.P. guns and 870 yards against 300 yards for towed guns).
(h) Out of a total of 83 actions it was found that the side which fired first was successful on 58 occasions and where one side had numerical superiority and fired first it was invariably successful.
(i) In tank versus tank engagements a successful attack was associated with losses of about 15% while failure was accompanied by losses of the order of 65%.
(j) Anti-tank gun losses in unsuccessful defensive actions were over 50% for S.P. guns and over 80% for towed guns while in successful defensive actions the losses were about 12½% and 15% respectively.

TANK BATTLE ANALYSIS.

PREPARED BY - Major G.Gee.
Major J.M.Murdoch Mitcheson.

SECTION I.

INTRODUCTION AND OBJECT.

1. During the war, operational research sections first in the MIDDLE EAST and later in N.W.EUROPE attempted mathematical analyses of armoured actions. Initially the object of these was to obtain overall figures for the numerical superiority required for various (mathematical) "chances of success" in much the same manner as the "chance of success" has long been calculated in anti-aircraft and field gunnery. Such studies were concerned mainly with tank v tank actions although a certain amount of work was done on tank v gun actions. Standard statistical rules for evaluating and determining the significance of the figures were used. The outcome of this work was promising in that there was strong indication of quantitative proof of such matters as the superiority of S.P. anti-tank guns compared with towed guns. Results however, from the statisticians point of view were far from sound because figures were incomplete. Consequently, at the end of the war the then Scientific Adviser to the Army Council ordered a more thorough investigation to be carried out using all available information. War diaries were found to be the best main source, (supplemented by commanders reports, dispatches etc) and examination of these yielded sufficient detail of some 200 small scale actions.

2. This report presents the method and results of the analysis. The object is to show how the chances of success in battle depended upon certain primary factors (e.g. numerical superiority). Analysis was attempted of a large number of these factors, but many had to be discarded in the course of the study. Those that remain are:-
(a) Numbers of tanks (or anti-tank guns) (i.e. numerical superiority).
(b) Types of tanks (or anti-tank guns) (i.e. quality of weapon).
(c) Ranges of effective engagements.
(d) Effect of firing first.
Among those discarded may be mentioned infantry support and surprise. No significant results could be obtained for the effect of these. On the other hand "Efficiency Factors" for the principal weapons, Allied and enemy, have been calculated. These express the relative superiority of one type of tank (or gun) over another. The analysis is restricted to small scale actions, arbitrarily defined as those in which less than 15 tanks were engaged -- mainly because it is in these that such factors as range of engagement are clearly defined, times of engagement are short, and results conclusive. In large scale actions this is not generally so. The battle is directed by commanders who do not form part of a "tank-plus-crew" unit actually engaged, and the number of such battles is too small for generalizations to be made (in the statistical sense) from them.

3. It should be emphasized at the outset that all the conclusions in this report are "overall", in which the effects of the manifold variables affecting tank fighting are averaged out. This is best understood by an example. The "efficiency factor of Allied S.P. guns over towed guns is given as 3:1. This means that averaged over a number of actions, and measured by success in battle only three towed gun-plus-crews were as good as one S.P. gun-plus-crew. All the many diverse factors such as speed, hitting power, rate of fire, crew morale, tactical skill, conditions of going and light are averaged out, and no conclusion concerning any one of these separately can be deduced. Also "out of battle" factors of merit such as reliability, bridge-crossing capacity, weight (for transportation) do not enter into the calculations at all, neither do such questions as manufacturing costs, relative time and cost of selection and training of crews etc. Thus the efficiency factor of 3 in favour of S.P. guns quoted here does not constitute implicitly an out-and-out recommendation that towed anti-tank guns should be abolished in favour of S.Ps. It is a measure of the weapons-plus-team's relative chances of success in battle during the last war in small scale action only, as reported in war diaries etc. In fact, "chance of success" in turn is generally a measure of casualty producing power.

Method.

4. The sources of information from which the figures were collected are given in Appendix A, the numbers in the brackets giving the total number of actions from each source. A large number of tank and anti-tank gun engagements are described in these sources. These have been studied and, as far as possible, the following information extracted in respect of each.

(i) The number of Allied tanks or guns involved.
(ii) " " " " " " " destroyed.
(iii) The number of Enemy tanks or guns involved.
(iv) " " " " " " " destroyed.
(v) Which side attacked.
(vi) Which side was successful.
(vii) Which side fired first.
(viii) Whether or not there was infantry support.
(ix) Range of engagement.

This data has been tabulated at Appendix 'B'.

5. Appropiate groups of actions have been selected from this table and analysed in order to:-

(a) compare similar weapons on opposite sides e.g. German and Allied tanks.
(b) compare two types of weapon on the same side e.g. Allied S.P. gun and Allied towed gun.
(c) evaluate the mean range at which tanks and anti-tank guns were used successfully.
(d) determine the relation, if any, between the chances of success and the ability to fire first.
(e) determine, if possible, the percentage losses associated with success and failure for the defence.

LIMITATIONS OF THE REPORT.

6. Although a large number of actions are described in the sources referred to in para. 4 only a small proportion of them are recorded in sufficient detail to be of any use for our purpose. The lack of detail was particularly noticeable in War Diaries where small-scale actions are often dismissed with a few words, no mention being made of even the number of tanks and /or anti-tank guns involved.

Consequently the analyses listed in para 5. have been made on groups of actions varying in size from 16 to 30 (except in the case of Allied S.P. guns versus enemy tanks where 66 actions were available) and these examples are obviously too small to permit of firm conclusions being drawn.

7. The types of weapons involved in the actions considered in this report were as follows:-

(a) Tanks.
Allied - Sherman 75 mm. Cromwell 75 mm.
Sherman 17 pdr. Challenger 17 pdr.
German - PZKW Mk.III, Mk.IV, Mk.V. Mk.VI.

(b) Anti-tank guns.
Allied S.P. 75 mm.
Allied towed guns 17 pdr and 6 pdr.
German S.P. 75 mm and 88 mm.
German towed guns 75 mm and 88 mm.

For brevity the expressions Allied tanks, German S.P. guns etc, have been used throughout this report. These expressions must be taken to refer to the types of weapons listed above. Unfortunately it was not possible to distinguish between Allied 17 pdr. and 6 pdr. towed anti-tank guns nor between German 75 mm. and 88 mm. guns. It is realised that this does, to some extent, detract from the value of some of the conclusions which are drawn, particularly those concerning Allied towed guns where the calibres of the two types differ greatly. However, the difficulty is an inescapable one and we can do no more than point it out. It may be noted that, in the sample considered, about 75% of the Allied towed guns were 17 pdr. and about 25% were 6 pdr; the German guns were roughly 50% 75mm and 50% 88 mm.

8. Since the only information available was from Allied sources it follows that:-
(a) the number of enemy successes recorded is small and lacking in detail.
(b) the figures given for numbers and types of enemy weapons and range at which used are only estimates which cannot be considered wholly reliable.

9. For the reasons given above the conclusions reached in this report must be regarded as tentative and subject to revision if and when more data become available.

SECTION II.

DEFINITIONS.

Criterion for Success.

10. It was realised that actions which were termed a success for either side in war diaries may only have been so in the opinion of the diarist. For one side to lose more tanks, numerically, in combat compared with the other side, is not always a sign of defeat, nor is a withdrawal of the Enemy conclusive proof of an Allied success. In the few doubtful cases which arose therefore, the side which suffered the least percentage casualties was deemed to be successful for the purpose of this report. It will be seen that this criterion is implied in the method of analysis used.

Efficiency Factor.

11. This is most easily defined by means of an example. Suppose that for a number of actions between Allied S.P. guns and German tanks the ratio

$$\frac{\text{Allied S.P. gun \% casualties}}{\text{German tank \% casualties}} = x$$

and that for a number of actions between Allied towed A/Tk. guns and the same type of German tank the ratio

$$\frac{\text{Allied towed gun \% casualties}}{\text{German tank \% casualties}} = y.$$

Then the Efficiency Factor of the S.P. gun over the towed A/Tk. gun is defined as

$$\frac{y}{x} = E \text{ say.}$$

In other words one S.P. gun is equivalent to E towed guns in battle.

Balancing Factor.

12. Again this is best defined by an example.

Suppose that for a number of actions between Allied tanks and German S.P. guns the number of Allied successes is "a" and the number of enemy successes is "e". For the "a" actions in which the Allies were successful let the average ratio

$$\frac{\text{number of Allied tanks engaged}}{\text{number of German guns engaged}} = X$$

For the "e" actions in which the Germans were successful let the average ratio

$$\frac{\text{number of German guns engaged}}{\text{number of Allied tanks engaged}} = Y$$

The difference between X and Y must be significant if there is any real difference between the efficiencies of the weapons being considered. The Balancing Factor is defined as the mean of X and $1/Y$ (where X and Y are significantly different) i.e.

$$\frac{X + 1/Y}{2}$$

This Factor is that ratio of numbers which gives an equal chance of success to either side and it is therefore a measure of the relative efficiencies of the two weapons.

SECTION III.

EFFICIENCY FACTORS.

13. The calculations for the various Efficiency Factors, and their significance tests are contained in Appendix C. The following table is a summary of the results obtained.

Table 1.

Type of factor.	No. of actions considered.	Efficiency Factor.	Significance test.	Remarks.
Allied 75 mm.S.P.gun over Allied 17 pdr and 6 pdr towed guns	64	3	5.4	Significant.
Enemy 75 mm. & 88 mm. S.P. guns over Enemy 75 mm. and 88 mm.towed guns.	41	2	49	Significant.
Enemy Mk.V & Mk.VI tanks over Enemy MK.III & Mk.IV tanks (versus Allied 75 mm. and 17 pdr. tanks).	43	1	.4	Not Significant.
Enemy Mk.V and Mk.VI tanks over Enemy Mk.III & Mk.IV tanks (versus Allied 75 mm. S.P.guns)	27	4	3.6	Significant.

Allied S.P. and towed A/Tk guns.

14. The Efficiency Factor of 3 for Allied S.P. Guns over towed guns is significant and since it is derived from a fairly large number of actions it may be accepted as correct within small limits of error.

15. Undoubtedly a large number of casualties to towed guns were caused by loss of gun personnel due to mortar and S.A.fire. On the other hand numerous instances are quoted in War Diaries of S.P. guns being used in a tank role and this may have resulted in their casualty rate being unnecessarily high.

16. A clear example of the superiority of S.P. guns over towed guns is provided by the figures for one British A/Tk regiment covering the period June - December 1944. Of the 20 enemy tanks claimed during that period 15 were knocked out by S.P. guns as against 5 by the towed guns of the regiment. The regiment's losses were 16 towed guns and 3 S.P. guns.

17. The following points are noted from the accounts appearing in war diaries:-

(a) many towed guns became casualties because of infantry withdrawal, after the retirement of the gun/tractors. (Firing mechanisms were invariably removed.

(b) Towed guns were often successful when manhandled into alternative positions during action, after one or more of the troop had been knocked out.

(c) A number of instances occur (see No.1 Operational Research Section report No.19) of M.10s being abandoned or destroyed due to danger of capture by Enemy infantry.

(d) Quite often an S.P. gun was called upon to deal with an Enemy tank, when an Allied tank armed with a 75 mm. gun had proved ineffective.

Enemy S.P. and Towed A/Tk guns.

18. The results for Enemy A/Tk. guns versus Allied tanks, also show a superiority of the S.P. guns over towed guns. The value of 2, so obtained from the analysis of 41 actions, has a very high significance figure and may therefore be accepted.

19. It is stressed in many war diary accounts of tank actions, that invariably, German towed A/Tk guns were extremely well concealed, and when encountered, the element of surprise was usually complete. This in itself would help to minimise the number of towed gun casualties, and is probably responsible for the shorter ranges at which these guns were used. This point is discussed subsequently.

Enemy tanks.

20. The value for the Efficiency Factor (1) of German Mk.V and VI over Mks.III and IV is not significant when the calculation is made from the actions of Allied and Enemy tanks. (App.C para 3 (a)). It is more than likely that this result is affected by the inclusion in the former class (Mks.V and VI) of casualties to Mk.IV Specials, which have been erroneously classified as Panthers in the written accounts. The similarity between the Mk.IV Special and Mk.V was encountered in most War diaries. This resulted in a claim being made that a Panther had been knocked out, and later it was confirmed that the Enemy casualty had been a Mk.IV.

21. In analysing the actions between German tanks and Allied S.P. guns, an efficiency Factor of 4 is obtained for Mks.V and VI over Mks. III and IV. This value is significant, although it would appear to be too high as a comparison between the two classes of German tanks. It does however indicate that the Mks.V and VI classed together were more effective against S.P. guns at least, than were the Mks.III and IV when classed together.

SECTION IV.

ALANCING FACTORS.

2. At Appendix D will be found a set of tables (compiled from Appendix) which show, for various types of opposed weapons, the average ratio of trengths (a) in all cases of Allied successand (b) in all cases of Enemy uccess. The difference between the ratios for each type of weapon has been ested for significance. The results of these calculations are summarised in able II below.

TABLE II

ype of action.	(a) Ratio of numbers Allied: Enemy, for ALLIED SUCCESS ONLY	(b) Ratio of numbers Enemy: Allied for ENEMY SUCCESS ONLY.	(c) Sign-nifi-cance Test.	Remarks.
i)Allied 75mm.& 17 dr tanks versus nemy Mk.III,Mk.IV, k.V & Mk.VI tanks.	1.6	1.1	4	significant.
ii)Allied 75mm. & 7pdr tanks,versus nemy 75mm.& 88mm. .P. and towed guns.	1.8	0.6	4.4	significant.
iii)Allied 75mm. .P. gun & 17pdr & pdr towed guns ersus Enemy Mk.III, k.IV,Mk.V & Mk.VI anks.	1.8	2.1	1.3	not significant.

llied Tank and Enemy Tank.

3. Serial I of Table II gives a Balancing Factor of 1.3 in favour of erman tanks. That is to say in an engagement where Allied tanks out-number heir opponents by 1.3 to 1 the chances of success will be equal for either ide. It is considered that this figure may, to some extent, exaggerate the he superiority of German tanks since in 70% of the actions considered the llies were the attackers and they did have a marked numerical superiority. n other words the sample is an "unbalanced" one.

llied Tank and Enemy A/Tk Gun.

4. In this case the Balancing Factor is 1.8, again in favour of the erman weapon. This factor is much more closely defined than the one for llied and German tanks, being derived from the two ratios 1.8 and 1.7 com-ared with 1.6 and 1.0 in the case of the tanks. It is worthy

of note that this factor of I.8 while showing the definite superiority of the German A/Tk gun over the Allied tank does not fully bear out the very high reputation which the German 88 mm A/Tk gun enjoyed in the eyes of our troops.

Allied A/Tk guns and Enemy Tanks.

25. No Balancing Factor can be calculated for these weapons because the ratios in columns (a) and (b) of Table II are not significantly different. This is probably due to the small number of enemy successes recorded (only 9) so that the sample is not typical. The upper (?) limit of I.8 is probably defined with fair accuracy since this figure is derived from 50 actions.

RELATION BETWEEN RANGE AND SUCCESS.

26. The following table shows the average range at which tanks and anti-tank guns were successful against a given opponent. Details of the calculation of these average ranges and their standard deviations will be found in Appendix E.

Table III.

Mean ranges at which tanks and guns were used successfully against various opponents.

(1) Serial No.	(2) Weapon	(3) Opponent	(4) Mean Ranges (yds)	(5) No. of Tanks or guns	(6) Standard Deviation
1.	Allied 75 mm. tank	German tank (MK111 & 1V)	790	20	710
2.	Allied 75 mm. tank	German tank (MK.V & V1)	390	23	322
3.	Allied 17 p.dr. tank	German tank (all types)	1100	22	719
4.	Allied 17 pdr tank	German S.P. gun	1270	10	1512
5.	Allied 17 pdr tank	German towed gun	455	11	89
6.	Allied tanks (all types)	German tanks (all types)	750	65	674
7.	Allied S.P. gun	German tanks (all types)	1090	34	715
8.	Allied towed A/TK gun (6 pdr & 17 pdr)	German tanks (all types)	870	30	414
9.	German tanks (all types)	Allied tanks (all types)	1290	27	702
10.	German S.P. gun	Allied tanks (all types)	330	7	195
11.	German towed A/TK gun	Allied tanks (all types)	300	14	214

The figure in column 4 of this table is the average range at which the weapon named in column 2 was successful against the one named in column 3. In column 5 is entered the number of ranges from which the mean was calculated.

27. We can now compare:-

(a) the success ranges of the same weapon against different opponents e.g. the Allied 17 pdr. tank against the S.P. gun and against the towed A/TK. gun.

(b) the success ranges of two weapons of the same side against the same opponent e.g. Allied 75 mm. tank against German tanks and Allied 17 pdr. tanks against German tanks.

and German S.P. guns against Allied tanks.

This has been done and the differences tested for significance. Details of the calculation will be found in Appendix E and the results are summarised below in Table IV.

Table IV.

Comparison of mean success ranges.

(1) Serial No.	(2) Weapon	(3) Opponent	(4) Mean Range (yds)	(5) Difference	(6) Significance Test.	(7) Remarks
1.	Allied 75mm. tanks	German tanks Mks. III & IV	790	400	3.5	Probably significant.
		German tanks Mks. V & VI.	390			
2.	Allied 17 pdr. tanks.	German S.P. gun	1270	815	2.5	NOT significant.
		German towed gun	455			
3.	Allied 75mm. tanks	German tanks (all types)	580	520	4.4	Significant
	Allied 17 pdr. tanks.		1100			
4.	Allied S.P. gun	German tanks (all types)	1090	220	2.3	NOT significant
	Allied towed gun		870			
5.	German S.P. gun	Allied tanks (all types)	330	27	0.5	NOT significant
	German towed gun		300			
6.	Allied tanks (all types)	German tanks (all types)	750	540	5.1	Significant
	German tanks (all types)	Allied tanks (all types)	1290			
7.	Allied S.P. guns	German tanks (all types)	1090	760	8	Significant
	German S.P. guns	Allied tanks (all types)	330			
8.	Allied towed guns	German tanks (all types)	870	570	6	Significant
	German towed guns	Allied tanks (all types)	300			

Tank v. Tank actions.

28. From serial 1 it will be seen that Allied 75 mm. tanks were successful against German Mk.III and IV tanks at nearly twice the range at which they were successful against German Mk.V and VI tanks. It cannot be said definitely that this difference is significant but it does indicate a difference between the two classes of German tanks.

29. Serial 3 shows clearly the greater effective gun-range of the Allied 17 pdr. tank compared with the 75 mm.

30. Serial 6 shows that German tanks were successful against Allied tanks at significantly greater range than vice versa.

Tank v. A/Tk. gun actions.

31. The mean success ranges of S.P. and towed guns (both German and Allied) do not differ significantly (serials 4 & 5).

32. Allied anti-tank guns (both S.P. and towed) were successful at significantly greater ranges than the corresponding German weapons (serials 7 & 8).

SECTION VI.

EFFECT OF FIRING FIRST.

33. The following table has been extracted from Appendix B in order to show the association between success in an action and the ability to fire first. In order to complete the picture the table also shows the numbers of tanks or guns engaged on either side. It will be seen that actions in which one side had an excessive numerical superiority have been excluded.

Table V.

------ Ratio of -----			
No. of tanks or guns on side which fired first.	No. of tanks or guns on other side.	Successes of side that fired first.	Failures of side that fired first.
1	4	2	-
1	3	4	8
2	5	1	1
1	2	6	5
2	3	4	4
3	4	1	1
1	1	20	5
2	1	8	1
3	2	1	0
4	3	2	0
3	1	7	0
4	1	2	0
	Totals	58	25

34. Thus in 58 out of the 83 actions listed, the side that fired first was also successful. The degree of association between the side that fires first and its success is particularly noticeable, as might be expected, in 1 to 1 engagements.

35. The distribution of failures in the sample is of interest in that the following conclusions can be drawn from them.

(a) The side with numerical superiority, which fired first, was invariably successful.

(b) The side with numerical inferiority, which fired first, was not always successful. Where this numerical inferiority was 2 to 3 the chances of success were even, but these deteriorated as the difference between the numbers on each side increased.

36. Naturally, the first shot fired may not always knock out a tank, and from the figures available, it was not possible to determine the effect of a "first miss". The ability to fire first, so closely related to superior tactical ability and gun handling does however play a paramount part in the outcome of an action.

ATTACKERS AND DEFENDERS LOSSES.

37. The tables in Appendix F, showing the losses of attackers and defenders in various types of action, were based upon the data in Appendix B. Since the numbers engaged on either side has a direct influence on the success or failure, actions in which either side had an excessive numerical superiority have been excluded.

38. The results obtained in Appendix F are summarised in Table VI below.

Table VI.

Losses sustained by Tanks and A/Tk guns.

Tank v. Tank actions.		
Attackers	Attackers losses	
	Successful attacks	Unsuccessful attacks
Allied tanks	10%	64%
Enemy tanks	20%	66%
Average	15%	65%

Tank v. A/Tank gun actions		
Defenders	Defenders Losses.	
	Successful Defence	Unsuccessful Defence
Allied S.P. guns	12%	54%
Enemy S.P. guns	13%	100%
Average	12½%	77%
Allied towed guns	20%	84%
Enemy towed guns	9%	100%
	15%	92%

39. As will be seen from Appendix F the number of actions available in the majority of cases is not large enough to give a representative result. It is not easy therefore to make deductions which can be taken as wholly reliable and the points made in the following paragraphs must be regarded as tentative only.

40. In tank versus tank actions a successful attack is associated with losses of about 15% while failure is accompanied by losses of the order of 65%.

41. Anti-tank gun losses in unsuccessful defensive actions are over 50% for S.P. guns and over 80% for towed guns. In the case of S.P. guns it is not clear whether, with half their guns gone, the defenders are physically unable to stop the advance or whether they tend to retire. The higher losses of towed guns compared with S.P. guns reflects the comparative immobility of the former and hence the difficulty of withdrawing them.

SUMMARY OF CONCLUSIONS.

42.

(a) S.P. guns are more effective than towed A/Tk guns, the Efficiency Factors being 3 in the case of Allied weapons and 2 for German weapons.

(b) German Mark V and VI tanks were more effective than Mark III and IV tanks against Allied S.P. guns by a factor of 4.

(c) In tank versus tank engagements, for the chance of success to be equal for either side, Allied tanks would have to outnumber the German tanks by some 30% or about one third.

(d) In an engagement between Allied tanks and German A/Tk guns the tanks would have to outnumber the guns by 2 to 1 for the chance of success to be equal for either side.

(e) The mean "success range" of the Allied 17 pdr tank against German tanks was nearly twice that of the Allied 75 mm. tank (1100 yards as against 580 yards).

(f) In tank versus tank engagements German tanks were successful at an average range of 1290 yards whereas Allied tanks were successful at an average range of only 750 yards.

(g) Allied A/Tk guns, (both S.P. and towed guns), were successful against tanks at significantly greater ranges than were the corresponding German weapons (1090 yards against 330 yards for S.P. guns and 870 yards against 300 yards for towed guns).

(h) Out of a total of 83 actions it was found that the side which fired first was successful on 58 occasions and where one side had numerical superiority and fired first it was invariably successful.

(i) In tank versus tank engagements a successful attack is associated with losses of about 15% while failure is accompanied by losses of the order of 65%.

(j) A/Tk gun losses in unsuccessful defensive actions were over 50% for S.P. guns and over 80% for towed guns while in successful defensive actions the losses were about 12½% and 15% respectively.

for Colonel
Superintendent.
M.O.R.U.

APPENDIX A.

Sources of Information.

The following are the sources of information from which the actions were compiled. The numbers in the brackets give the total actions from each source.

War diaries of 31 armoured units in B.L.A. during June 44 to May 45. (35)

No.2 Operational Research Section report No.27 on "Anti-tank guns in the Artillery". (38)

No.1 Operational Research Section report No.5 on "Notes on a visit to the 4th Armoured Brigade in Italy in Nov.43". (14)

Studies by Lt.Col. Whittaker on 3 actions in Tunisia during April and May 43. (4)

Operational Research Memo No.4 GHQ/MEF/SD6, "Report on a tour of Armoured Formations in North Africa, April and July 43". (2)

War diaries of 25 Armoured units in Italy, Jan 44 to May 45. (29)

War diaries of 12 Canadian Armoured units in Italy from Jan 44 to May 45, and in the B.L.A. from June 44 to May 45. (15)

7 US Army files (approx 100 unit-months records). (11)

No.1 Operational Research Section report No.19. (22)

War diaries of 20 anti-tank units in B.L.A. from June 44 to May 45, and in the C.M.F. from Jan 44 to May 45. (30)

APPENDIX B -1-

SUMMARY OF ACTIONS.

Type of Action.	No. of Allied Tks.	Allied casualties	No. of Enemy Tks.	Enemy casualties	Attackers	Success	Who fired first	With Inf. support	Range in yds.
1. TANK v. TANK.									
Sherman 75)	?4	2	2	2	?	A	?		200
Cromwell 75) v	1	0	1	1	A	A	A		2200
-(Mk. III	2	0	1	1	A	A	A		1700
(Mk. IV	2	0	1	1	A	A	A		1600
	1	0	1	1	A	A	?		1800
	1	0	2	1	A	A	A		1500
	1	0	1	1	A	A	A		15
	3	0	1	1	E	A	A	A-E	200
	1	0	1	1	E	A	A		?
	1	0	1	1	A	A	A		?
	1	1	1	0	E	E	A		?
	1	1	2	1	A	E	?		?
	5	2	2	2	A	A	E		300-600
	1	1	1	0	A	E	E		?
	3	2	1	0	A	E	?		?
2. TANK v. TANK.									
Sherman 75)									
Cromwell 75) v.	1	0	1	1	A	A	A		700
-(Mk. V	4	0	2	2	A	A	A		800
(Mk. VI	3	0	1	1	A	A	A	A	100
	4	0	2	2	A	A	A	A	800
	2	1	1	1	A	A	E	E	30
	1	0	1	1	E	A	A		150
	1	0	1	1	A	A	A	A-E	150
	3	0	1	1	A	A	A	A	150
	4	0	2	1	A	A	E		200
	8	2	?	2	A	?	?		?
	1	0	2	1	A	A	A		?
	3	1	1	1	A	A	E		?
	1	0	2	1	?	A			?

SUMMARY OF ACTIONS (Contd).

Type of Action.	No. of Allied Tks.	Allied casualties	No. of Enemy Tks.	Enemy casualties	Attackers	Success	Who fired first	With Inf. support	Range in yds.
TANK v. TANK (Contd).									
	1	1	1	0	A	E	A		100
	3	2	2	0	A	E	E		30
	3	2	1	0	A	E	E	E	150
	1	1	1	0	A	E	E		?
	1	1	2	1	E	E	A		?
TANK v. TANK.									
erman & Challenger	2	0	1 (Mk. IV)	1	E	A	E		200
pdr tanks.	7	1	5 (Mk. V)	3	E	A	?		?
ctures of German	4	2	3	0	E	E	E		?
ks.	15	0	4	2	E	A	?		1100
	5	?	2	2	E	A	A		1000
	5	0	5	4	A	A	A		800
	?30	0	4(2=Mk. IV) (2=Mk. V)	2	E	A	?		1000
	1	0	1 (Mk. IV)	1	A	A	A		2400
	3	0	3 (Mk. V)	2	A	A	A		150
	6	0	6(1=Mk. VI) (5=Mk. V)	3	E	A	A		2000
	1	0	1 (Mk. V)	0	E	?	A		?
	1	0	1 (Mk. VI)	1	A	A	A		?
	2	0	1 (Lt. Tk.)	1	?	A	A		?
	4	2	4(3=Mk. IV, 1 Mk. V)	1	?	E	?		1000
	?40	8	8(Mk. III, IV, VI)	7	A	A	?		?
	3	2	4 (Mk. V, VI)	1	E	E	E		1000
	2	0	3 ?	0	A	E	A		800
	7	4	12	?	A	E	E		2000
	3	0	1 (Mk. VI)	0	A	?	?		?
	3	0	3 (Mk. VI)	0	A	?	A		?

SUMMARY OF ACTIONS (Contd).

Type of Action.	No. of Allied Tks. or guns	Allied casualties	No. of Enemy Tks. or guns	Enemy casualties	Attackers	Success	Who fired first	With Inf. support	Range in yds.
4. TANK v. S.P.A/TANK.									
Sherman & Challenger	4	1	1	0	A	E	E		200
17 pdr tanks v.	1	0	1	1	A	A	A		3000
German 75 mm and	1	0	1	1	A	A	A		2000
88 mm S.P. guns.	1	0	1	1	A	A	A		3000-4000
	?	0	4	4	E	A	?		?
	10	7	5	2	A	E	E		?
	6	4	3	0	A	E	E	E	?
	1	0	1	1	E	A	A		?
	1	1	1	0	A	E	E		20
	4	4	2	0	A	E	E		?
	2	1	1	0	A	E	E		200
	6	2	3	0	A	E	?	E-A	?
	5	3	1	0	A	E	E		?
	1	0	1	1	A	A	?		1500
	1	0	1	1	A	A	?		200
	1	1	1	0	A	E	A		?
	1	1	1	0	A	E	E		?
	2	0	1	0	E	?	E		?
	3	2	2	1	A	E	E		500
	2	0	1	1	A	A	E		200
	3	1	1	1	A	A	E	A	700
	3	1	1	0	A	E	E	A	250
	3	1	1	0	A	E	E	A	600
5. ALLIED TANK v. ENEMY TOWED A/TK GUN.									
Sherman & Challenger	15	1	1	1	A	A	E		4700
17 pdr tanks v.	15	0	1	1	A	A	A		?
German 75 mm and	?4	1	7	0	A	E	?		300
88 mm towed guns.	19	0	7	7	A	A	?		?
	6	3	1	1	A	A	E		500
	17	4	1	1	A	A	E		1000

SUMMARY OF ACTIONS (Contd).

Type of Action.	No. of Allied Tks. or guns	Allied casualties	No. of Enemy Tks. or guns	Enemy casualties	Attackers	Success	Who fired first	With Inf. support	Range in yds.
5. ALLIED TANK v. ENEMY TOWED A/TK GUN (Contd)									
	3	0	2	0	A	?	E	A	?
	15	5	1	1	A	A	E		200
	3	0	1	1	A	A	E		?
	3	0	1	1	A	A	E		?
	1	0	1	1	A	A	E		?
	3	0	2	2	A	A	E		500
	1	0	2	2	A	A	?		?
	1	1	1	0	A	E	E		200
	2	2	1	0	A	E	E		?
	1	0	1	1	A	A	E		?
	1	0	2	2	A	A	E		200
	2	0	1	1	A	A	A		400
	4	2	1	0	A	E	E		100
	3	2	1	1	A	A	E		60
	3	2	1	0	A	E	E	A	1000
	3	1	2	1	A	E	E		100
	6	3	1	0	A	E	E	E	400
	4	1	1	0	A	E	E	A-E	200
6. A/TANK v. TANK.									
Allied 75 mm S.P. gun v. Enemy tanks - all types.	12	3	?	14	E	A		A	
	12	6	?	16	E	A		A	
	1	0	2	2	E	A			
	4	4	?	0	E	[illegible]			
	24	9	?	3	E	A		A	
	4	0	?	3	E	A		A	
	2	0	2	1	E	A		A-E	
	1	0	?	2	E	A		A	
	12	2	?	5	E	?		A-E	
	?12	6	?	9 @	E	E		A-E	@ also S.P. destroyed.
	? 8	0	?	2	E	A		A	
	4	0	25	4	E	E		A-E	

SUMMARY OF ACTIONS (Contd).

Type of Action.	No. of Allied Tks. or guns	Allied casualties	No. of Enemy Tks. or guns	Enemy casualties	Attackers	Success	Who fired first	With Inf. support	Range in yds.
A/TANK v. TANK (Contd).									
	4	0	?	4 @	E	A		A	@ also 1 Half-track
	?8	0	?	2	E	A		A	
	4	0	?	1	E	A		A	
	4	3 @	?	6	E	A		A-E	@ all by arty.
	36	6	?	27	E	A		E	
	36+	0	30-40	2	E	A		E	
	4	0	8	1	E	A		A-E	
	4	0	16	1	E	A		A-E	
	4	0	3-4	1	E	A		A-E	
	2	2	16	14	E	A		A-E	E. Inf. successful
	4	1	7	3	E	A		A	
	2	0	6	3	E	A	A		600-800
	4	0	1	1	?	A	A		3000
	4	0	3	2	?	A	?		?
	4	0	3	1	E	A	E		?
	3	?	6	2	E	A	A	A	?
	4	0	1 (Mk. IV)	1	?	A	?		?
	4	1	1 (Mk. IV)	1	?	A	?		?
	3	1	5(3=Mk. VI, 2=Mk. IV)	2	?	A	?		Enemy with-drew
	4	1	6	1	?	E	?		
	4	1	4	2	?	A	?		
	3	3 @	2 (Mk. IV)	2	?	?	?	E	@ T.Ds. abandoned.
	2	1	12	3	?	?	?		
	4	0	2 (Mk. VI)	2	?	A	?		
	4	0	2 (Mk. VI)	2	?	A	?		
	4	4 @	1 (Mk. IV)	1	?	?	?		@ T.Ds. abandoned.
	4	1	5 (Mk. VI)	2	?	A	?		
	3	0	1 (Mk. VI)	1	?	A	?		
	4	0	3 (Mk. VI)	3	?	A	?		
	4	2	1 (Mk. VI)	1	?	A	?		
	4	0	2 (Mk. IV)	2	?	A	?	E	

SUMMARY OF ACTIONS (Contd).

Type of Action.	No. of Allied Tks. or guns	Allied casualties	No. of Enemy Tks. or guns	Enemy casualties	Attackers	Success	Who fired first	With Inf. support	Range in yds.
A/TANK v. TANK (Contd).									
	4	2	3 (Mk.VI)	1 @	?	E	?		@ 2 A/Tk. guns also
	3	0	1 (Mk.VI)	1	?	A	?		
	4	2	4 (Mk.VI)	4	E	A	?		
	4	0	1 (Mk.VI)	1	?	A	?		1000
	4	0	7 (Mk.VI)	1	E	A	?		1000
	4	0	5	3	?	A	?		
	4	0	3 (Mk.IV)	1	?	A	?		
	3	0	3 (Mk.VI)	2	?	A	A	A	900-1000
	3	0	2	1	?	A	A		900-1000
	2	0	3 (Mk.VI)	2	E	A	A		600
	3	?	2 (Mk.IV)	1	A	A	E		?
	2	2	1	0	A	E	E		?
	3	1	1	0	?	E	E		?
	3	0	1 (Mk.VI)	1	?	A	?		800
	3	0	1 (Mk.IV)	1	?	A	?		1000
	3	0	1 (Mk.IV)	1	?	A	?		600
	3	1	3	1	?	?	?		?
	3	0	1	1	E	A	A		?
	3	0 @	1 (Mk.VI)	1	A	A	A		? @ 1.S.P.
	3	1	1 (Mk.VI)	1	?	A	E		? K.Od. by
	3	1	?	3	E	?	?		1000 own Tp.
	3	0	?	3	E	?	?		1000
	3	0	1 (Mk.VI)	1	E	A	A		550
ALLIED TOWED ANTI-TANK GUNS v. ENEMY TANKS									
ied towed 17 pdr	3	2	3	3	E	A	A	E	?
pdr guns v. y tanks all s.	2	2	10	0	E	E		A-E	A Inf. inadequate
	8	0	6	1	E	A		A	
	12	0	?	3	E	A		A	

SUMMARY OF ACTIONS (Contd).

Type of Action.	No. of Allied Tks. or guns	Allied casualties	No. of Enemy Tks. or guns	Enemy casualties	Attackers	Success	Who fired first	With Inf. support	Range in yds.	
ALLIED TOWED ANTI-TANK GUNS v. ENEMY TANKS (Contd).										
	4	3 @	10	4 =	E	E		A-E		@ mines & art = Self destroyed
	4	4	?	0	E	E		E		
	12	8	?	4 @	E	E		A-E		@ also 1 half track
	8	8	?	1 O	E	E		A-E		@ also 2 S.Ps.
	4	4	?	14 @	E	E		A-E		@ also 1 S.P.
	2	2	?	0	E	E		A-E		
	6	6	?	6	E	E		A-E		
	2	1	?	0	E	E		A-E		
	2	2	?	0	E	E		E		
	2	2	?	1	E	E		E		
	?36	0	12	4	E	E		E		
	?36	31	?	11	E	?		E		
	8	2	9	4	E	A	E	A-E	1200	
	4	2	4	4	E	A	?	A	450	
	2	0	1 (Mk.VI)	1	E	A	A		700	
	4	0	3	2	E	A	A	A	800	
	4	1	10	3	E	A	?	A-E	?	
	2	2	6 (Mk.VI)	0	?	E	E		700	
	2	1 @	6	2	E	A	A	A	600 @	1 gun also
	2	1	3	2	E	A	A		500	abandoned
	1	0	20	9	E	A	A		200-400	
	4	1	1 (Mk.VI)	1	E	A	?		500	
	1	0	1 (Mk.V)	1	A	A	A		400	
	4	0	1	1	A	A	A		1600	
	4	2	1 (Mk.IV)	1	E	A	?		?	
	1	1	2	0	E	E	?		?	

SUMMARY OF ACTIONS (Contd).

Type of Action.	No. of Allied Tks. or guns	Allied casualties	No. of Enemy Tks. or guns	Enemy casualties	Attackers	Success	Who fired first	With Inf. support	Range in yds.
MIXED ACTIONS.									
8. ALLIED TANKS v. ENEMY TANKS & S.P. GUNS.									
	18	2	? (Mk.VI) S.P.	4 Tks. 2 S.Ps.	E	A		A-E	
	11	2	3 (Mk.IV) 2 S.P. 75	1 Tk.	A	E	E		1000-1700
	8	2	8 (Mk.IV) (S.P.) 2 S.P. 75	2 Tks.	E	E	E		800-2000
ALLIED TANKS & A/TK GUNS v. ENEMY TANKS.									
	3 4 towed guns	3 Tks. 1 gun	4 (Mk.IV)	4	E	A	E		200-400

EFFICIENCY FACTORS.

The following figures are taken from Appendix B. The figures which assume that 15 tanks are involved in what is reported as a Squadron action, have been omitted below, since the number 15 cannot be taken as definitely correct.

1. Comparison of Allied towed and S.P. A/Tk guns. (From App.B. 6 & 7)

(i) S.P. guns (75 mm.)
Actions where number of guns and tanks is known for both sides. (Totals from 46 actions).

No. of guns	155
Gun casualties	18
No. of Enemy tanks.	145
Tank casualties.	68

(ii) Towed guns. (17 pdr & 6 pdr.)
Totals from 18 actions.

No. of towed guns	60
Towed gun casualties	17
No. of Enemy tanks	97
Tank casualties.	35

From (i) $\frac{\text{S.P. gun \% casualties.}}{\text{Tank \% casualties}} = \frac{18}{155} \times \frac{145}{68} = .247$

From (ii) $\frac{\text{Towed gun \% casualties.}}{\text{Tank \% casualties}} = \frac{17}{60} \times \frac{97}{35} = .785$

Therefore Efficiency Factor of S.P. gun over towed gun $= \frac{.785}{.247} = 3$

	Allied gun % casualties	Ratio of gun to Enemy tank cas.	S.D.
Allied S.P. guns	11.6	.247	.13
" towed guns	28.3	.785	.077

Therefore p.c. $= 2/3\sqrt{(.13)^2 + (.077)^2} = .106$

Test for significance $= \frac{.785-.247}{.106} = 5.3$

2. Comparison of Enemy S.P. and towed A/Tk guns. (From App.B 4 & 5)

(i). S.P. guns (75 mm. & 88 mm.)
Actions where number of guns and tanks is known for each side. (Totals from 41 actions)

No. of guns.	32
Gun casualties.	11
No. of Allied tanks.	62
Tank casualties.	30

(ii) Towed guns (75 mm. and 88 mm).
Totals from 41 actions.

No. of guns.	30
Gun casualties.	14
No. of Allied tanks.	54
Tank casualties.	18

From (i) $\frac{\text{S.P. gun \% casualties}}{\text{Tank \% casualties}} = \frac{11}{32} \times \frac{62}{30} = .71$

From (ii) $\frac{\text{Towed gun \% casualties}}{\text{Tank \% casualties}} = \frac{14}{30} \times \frac{54}{18} = 1.4$

Efficiency Factor of Enemy S.P. gun over towed gun = $\frac{1.4}{.71}$ = 2

	Enemy gun % casualties.	Ratio of gun to Allied tank cas.	S.D.
Enemy S.P. guns	34.37	.71	.016
" towed guns	46.66	1.4	.014

p.c. = .016 and test for significance $\frac{1.4 - .71}{.016}$ = 43

3. Comparison of German Tanks.

(a) German tanks versus Allied tanks.

Totals from 43 actions.	(i) Mks.III & IV.	(ii) Mks.V & VI.
No. of Allied tanks (75 mm and 17 pdr.)	32	64
Allied tank casualties.	9	12
No. of enemy tanks.	23	48
Enemy tank casualties.	17	25

From (i) $\frac{\text{Allied \% casualties}}{\text{Mk.III \& IV \% casualties}} = \frac{9}{32} \times \frac{23}{17} = .38$

From (ii) $\frac{\text{Allied \% casualties}}{\text{Mk.V \& VI \% casualties}} = \frac{12}{64} \times \frac{46}{25} = .36$

Efficiency Factor of Mk.V & VI over Mk.III & IV = $\frac{38}{36}$ = 1

	Enemy % casualties.	Ratio of Allied to Enemy % casualties	S.D.
Mk.III & IV	73.9	.38	.056
Mk.V & VI	52	.36	.066

p.c. = .056 and test for significance $\frac{.38 - .36}{.056}$ = .35

(b) German tanks versus Allied S.P. guns.

Totals from 27 actions.	(i) Mk.III & IV.	(ii) Mk.V & VI.
No. of Allied S.P. guns (75 mm.)	32	56
Gun casualties.	1	6
No. of Enemy tanks.	14	39
Tank casualties.	11	26

From (i) $\frac{\text{Allied S.P. gun \% casualties}}{\text{Mk.III \& IV \% casualties}} = \frac{1}{32} \times \frac{14}{11} = .04$

From (ii) $\frac{\text{Allied S.P. gun \% casualties}}{\text{Mk.V \& VI \% casualties}} = \frac{6}{56} \times \frac{39}{26} = .16$

Efficiency Factor of Mks.V & VI over Mks III and IV. = $\frac{.16}{.04}$ = 4

	Enemy % casualties	Ratio of Allied to Enemy % casualties.	S.D.
Mks.III & IV	78.5	.04	.022
Mks.V & VI	66.6	.16	.044

p.c. = .04 and test for significance $\frac{.16 - .04}{.04.}$ = 3.0

BALANCING FACTORS.

1. Tank v tank actions. (Allied 75 mm. & 17 pdr. versus Enemy Mk.III, Mk.IV, Mk.V & Mk. VI tanks).

Tank v tank ALLIED SUCCESSES ONLY.

(a) Ratio of numbers Allied : Enemy	(b) Frequency of actions	(c) (a) x (b)	(d) Deviation from 1.6	(e) Square of Deviation	(f) (b)x(e)
.5 : 1	3	1.5	-1.1	1.21	3.63
1 : 1	14	14	-0.6	0.36	5.04
1.4 : 1	1	1.4	-0.2	0.04	0.04
2 : 1	9	18	+0.4	0.16	1.44
2.5 : 1	2	5	+0.9	0.81	1.62
3 : 1	4	12	+1.4	1.96	7.84
3.75: 1	1	3.75	+2.2	4.84	4.84
	34	55.65			24.45

Mean ratio of numbers = $\frac{55.65}{34} = 1.6$

Standard deviation = $\sqrt{\frac{24.45}{34}} = 0.84$

Tank v tank ENEMY SUCCESSES ONLY.

(a) Ratio of numbers Enemy : Allied	(b) Frequency	(c) (a) x (b)	(d) Deviation from 1.1	(e) Square of Deviation	(f) (b)x(e)
.3 : 1	2	.6	-0.8	0.64	1.28
.7 : 1	1	.7	-0.4	0.16	0.16
.8 : 1	1	.8	-0.3	0.09	0.09
1 : 1	5	5	-0.1	0.01	0.05
1.3 : 1	1	1.3	+0.2	0.04	0.04
1.7 : 1	1	1.7	+0.6	0.36	0.36
2 : 1	2	4	+0.9	0.81	1.62
	13	14.1			3.60

Mean ratio of numbers = $\frac{14.1}{13} = 1.1$

Standard deviation = $\sqrt{\frac{3.60}{13}} = 0.53$

Test for significance

Difference between above ratios: 1.6 - 1.1 = 0.5

p.e. = $\frac{2}{3}\sqrt{\frac{.84^2}{34} + \frac{.53^2}{13}} = .13$

$\frac{\text{Difference}}{\text{p.e.}} = \frac{0.5}{0.13} = 4$ and the difference is significant

2. Allied tank versus Enemy Anti-Tank gun actions. (Allied 75 mm. & 17 pdr. tanks versus Enemy 75 mm. and 88 mm. guns).

Allied tanks v Enemy A/Tk.guns. ALLIED SUCCESSES ONLY.

(a) Ratio of numbers Allied : Enemy.	(b) Frequency	(c) (a) x (b)	(d) Deviation from 1.8	(e) Square of deviation	(f) (b) x (c)
.5 : 1	2	1	-1.3	1.69	3.38
1 : 1	8	8	-0.8	0.64	5.12
1.5 : 1	1	1.5	-0.3	0.09	0.09
2 : 1	2	4	+0.2	0.04	0.08
2.7 : 1	1	2.7	+0.9	0.81	0.81
3 : 1	4	12	+1.2	1.44	5.76
6 : 1	1	6	+4.2	17.64	17.64
	19	35.2			32.88

Mean ratio = $\frac{35.2}{19}$ = 1.8

Standard deviation = $\sqrt{\frac{32.88}{19}}$ = 1.3

Allied tanks versus Enemy A/Tk.guns. ENEMY SUCCESSES ONLY.

(a) Ratio of numbers Enemy : Allied	(b) Frequency	(c) (a)x(b)	(d) Deviation from 0.6	(e) Square of deviation	(f) (b)x(c)
.2 : 1	2	0.4	-0.4	0.16	0.32
.3 : 1	6	1.8	-0.3	0.09	0.54
.5 : 1	6	3.0	-0.1	0.01	0.06
.7 : 1	2	1.4	+0.1	0.01	0.02
1 : 1	4	4.0	+0.4	0.16	0.64
1.8 : 1	1	1.8	+1.2	1.44	1.44
	21	12.4			3.02

Mean ratio = $\frac{12.4}{21}$ = 0.6

Standard deviation = $\sqrt{\frac{3.02}{21}}$ = 0.39

Test for significance

Difference between the two ratios = 1.2

p.e. = $\frac{2}{3}\sqrt{\frac{1.3^2}{19} + \frac{0.39^2}{21}}$ = .27

$\frac{\text{Difference}}{\text{p.e.}}$ = $\frac{1.2}{.27}$ = 4.4 and the difference is significant.

3. Allied Anti-tank gun versus Enemy tank.(Allied 75 mm. and 17 pdr. and 6 pdr. towed guns versus Enemy Mk.III, Mk. IV, Mk.V and Mk.VI tanks). Allied A/T k. guns versus Enemy tanks. ALLIED SUCCESSES ONLY

(a) Ratio of numbers Allied : Enemy	(b) Frequency	(c) (a)x(b)	(d) Deviation from 1.8	(e) Square of deviation	(f) (b)x(e)
.2 : 1	1	0.2	-1.6	2.56	2.56
.3 : 1	1	0.3	-1.5	2.25	2.25
.5 : 1	3	1.5	-1.3	1.69	5.07
.6 : 1	2	1.2	-1.2	1.44	2.88
.7 : 1	3	2.1	-1.1	1.21	3.63
.8 : 1	2	1.6	-1.0	1.00	2.00
.9 : 1	1	0.9	-0.9	0.81	0.81
1 : 1	9	9.0	-0.8	0.64	5.76
1.3 : 1	6	7.8	-0.5	0.25	1.50
1.5 : 1	2	3.0	-0.3	0.09	0.18
2 : 1	4	8.0	+0.2	0.04	0.16
3 : 1	9	27.0	+1.2	1.44	12.96
4 : 1	7	28.0	+2.2	4.84	33.88
	50	90.6			73.64

Mean ratio $= \frac{90.6}{50} = 1.8$

Standard deviation $= \sqrt{\frac{73.64}{50}} = 1.2$

Allied A/Tk. guns versus Enemy tanks ENEMY SUCCESSES ONLY.

(a) Ratio of numbers Enemy : Allied	(b) Frequency	(c) (a)x(b)	(d) Deviation from 2.1	(e) Square of deviation	(f) (b)x(e)
.3 : 1	1	0.3	-1.8	3.24	3.24
.5 : 1	1	0.5	-1.6	2.56	2.56
.8 : 1	1	0.8	-1.3	1.69	1.69
1.5 : 1	1	1.5	-0.6	0.36	0.36
2 : 1	1	2.0	-0.1	0.01	0.01
2.5 : 1	1	2.5	+0.4	0.16	0.16
3 : 1	2	6.0	+0.9	0.81	1.62
5 : 1	1	5.0	+2.9	8.41	8.41
	9	18.6			18.05

Mean ratio $= \frac{18.6}{9} = 2.1$

Standard deviations $\sqrt{\frac{18.05}{9}} = 1.4$

Test for significance

Difference between the two ratios = 0.4

p.e. $= \frac{2}{3}\sqrt{\frac{1.2^2}{50} + \frac{1.4^2}{9}} = 0.3$

$\frac{\text{Difference}}{\text{p.e.}} = \frac{0.4}{0.3} = 1.3$ and the difference is NOT significant.

Appendix E.

A. Calculation of mean success ranges and standard deviations.

1. Allied 75 mm. tanks (versus German tanks Mk.III and IV).

(a) Range in yards.	(b) No. of tanks.	(c) (a) x (b)	(d) Deviation.	(e) (Deviation)^2
15	1	15	-775	600625
200	7	1400	-590	348100
450	5	2250	-340	115600
1500	1	1500	+710	504100
1600	2	3200	+810	656100
1700	2	3400	+910	828100
1800	1	1800	+1010	1020100
2200	1	2200	+1410	1988100
Totals	20	15765		10096025

Mean success range = $\frac{15765}{20}$ = 750 yards.

Standard deviation = $\sqrt{\frac{10096025}{20}}$ = 710

Note: In the above table the figures in column (b) gives the number of Allied 75 mm. tanks which engaged successfully German Mark III and IV tanks at the range shown in column (a). The total entered in column (e) is the total of the squares of the deviations each multiplied by the corresponding frequency i.e. $\Sigma(b) \times (e)$. The tables given in the following paragraphs have been constructed similarly.

2. Allied 75 mm. tanks (versus German tanks Mark V and VI).

Range.	No. of tanks.	(Range x No.)	Deviation	(Deviation)^2
30	2	60	-360	129600
100	3	300	-290	84100
150	5	750	-240	57600
200	4	800	-190	36100
700	1	700	+310	96100
800	8	6400	+410	168100
Totals	23	9010		2384800

Mean success range = $\frac{9010}{23}$ = 390 yards.

Standard Deviation = $\sqrt{\frac{2384800}{23}}$ = 322

3. Allied 75 mm. tanks (versus German tanks all types).

This is a comination of paras. 1 and 2 above.

Mean success range = $\frac{15765 + 9010}{20 + 23}$ = 580 yards

Calculating deviations from this mean we get

Standard Deviation = $\sqrt{\frac{14163225}{43}}$ = 574

4. Allied 17 pdr. tanks (versus German tanks all types)

Range	No.	Range x No.	Deviation	(Deviation)^2
150	3	450	- 950	902500
200	2	400	- 900	810000
800	5	4000	- 300	90000
1000	5	5000	- 100	10000
2000	6	12000	900	810000
2400	1	2400	1300	1690000
Totals	22	24250		11377500

Mean success range = $\frac{24250}{22}$ = 1100 yards

Standard Deviation = $\sqrt{\frac{11377500}{22}}$ = 719

5. Allied Tanks (all types) (versus German tanks all types)

This is a combination of paras 3 & 4 above.

Mean success range = $\frac{(580 \times 43) + (1100 \times 22)}{65}$ = 750 yards

Standard Deviation = $\sqrt{\frac{29569525}{65}}$ = 674

6. Allied 17 pdr. tank (versus German S.P. gun)

Range	No.	Range x No.	Deviation	(Deviation)^2
200	3	600	- 1070	1144900
700	3	2100	- 570	324900
1500	1	1500	+ 230	52900
2000	1	2000	+ 730	532900
3000	1	3000	+ 2730	7452900
3500	1	3500	+ 3230	10432900
Totals	10	12700		22881000

Mean success range = $\frac{12700}{10}$ = 1270 yards

Standard Deviation = $\sqrt{\frac{22881000}{10}}$ = 1512

7. Allied 17 pdr. tank (versus German towed gun)

Range	No.	Range x No.	Deviation	(Deviation)^2
200	1	200	-255	65025
400	2	800	- 55	3025
500	8	4000	+ 45	2025
Totals	11	5000		87275

Mean success range = $\frac{5000}{11}$ = 455 yards

Standard Deviation = $\sqrt{\frac{87275}{11}}$ = 89

Allied S.P. gun (versus German Tanks all types)

Range	No.	Range x No.	Deviation	(Deviation)^2
550	3	1650	- 540	291600
600	5	3000	- 490	240100
700	2	1400	- 390	152100
800	3	2400	- 290	84100
950	6	5700	- 140	19600
1000	11	11000	90	8100
3000	4	12000	1910	3648100
Totals	34	37150		17430900

Mean success range = $\frac{37150}{34}$ = 1090 yards

Standard Deviation = $\sqrt{\frac{17430900}{34}}$ = 715

Allied towed A/Tk. guns (versus German tanks all types)

Range	No.	Range x No.	Deviation	(Deviation)^2
300	1	300	- 570	324900
400	1	400	- 470	220900
450	4	1800	- 420	176400
500	6	3000	- 370	136900
700	2	1400	- 170	28900
800	4	3200	- 70	4900
1200	8	9600	+ 330	108900
1600	4	6400	+ 730	532900
Totals	30	26100		5153000

Mean success range = $\frac{26100}{30}$ = 870 yards

Standard deviation = $\sqrt{\frac{5153000}{30}}$ = 414

German tanks (all types)(versus Allied tanks all types)

Range	No.	Range x No.	Deviation	(Deviation)^2
30	2	60	- 1260	1587600
100	1	100	- 1190	1416100
150	1	150	- 1140	1299600
800	3	2400	- 490	240100
1000	8	8000	- 290	84100
2000	12	24000	+ 710	504100
Totals	27	34710		13333200

Mean success range = $\frac{34710}{27}$ = 1290 yards

Standard Deviation = $\sqrt{\frac{13333200}{27}}$ = 702

Range	No.	Range x No.	Deviation	(Deviation)^2
20	1	20	- 310	96100
200	2	400	- 130	16900
250	1	250	- 80	6400
500	2	1000	+ 170	28900
600	1	600	+ 270	72900
Totals	7	2270		267000

Mean success range = $\frac{2270}{7}$ = 330 yards

Standard Deviation = $\sqrt{\frac{267000}{7}}$ = 195

12. German towed A/Tk. gun (versus Allied tanks all types)

Range	No.	Range x No.	Deviation	(Deviation)^2
100	3	300	-200	40000
200	2	400	-100	10000
300	7	2100	0	0
400	1	400	+100	10000
1000	1	1000	+700	490000
Totals	14	4200		640000

Mean success range = $\frac{4200}{14}$ = 300 yards

Standard Deviation = $\sqrt{\frac{640000}{14}}$ = 214

B. Comparison of mean success ranges

1. Success range of Allied 75 mm. tank against German Mk. III & IV tanks and against German Mark V & VI tanks.

Opponent	Success Range	S.D.	n.
Mk. III & IV tanks	790	710	20
Mk. V & VI tanks	390	322	23
Difference	400		

Probable error (p.e.) = $\frac{2}{3}\sqrt{\frac{710^2}{20} + \frac{322^2}{23}}$ = 115

Significance test:

$\frac{\text{Difference}}{\text{p. e.}} = \frac{400}{115} = 3.5$ ∴ Significant

Note: The number 'n' in the above table is the number of "observations" from which the mean range is derived. That is, it is the total of column (b) in the tables in Part A of this Appendix.

2. Success ranges of Allied 75 mm. tanks against German tanks (all types) and Allied 17 pdr. tanks against German tanks (all types)

Weapon	Success range	S.D.	n.
Allied 75 mm. tank	580	574	43
Allied 17 pdr. tank	1100	719	22
Difference	520		

$$\text{p.e.} = \frac{2}{3}\sqrt{\frac{574^2}{43} + \frac{719^2}{22}} = 117$$

Significance test: $\frac{\text{Difference}}{\text{p.e.}} = \frac{520}{117} = 4.4$ ∴ Significant

3. Success ranges of Allied tanks (all types) and of German tanks (all types) in tank v. tank actions

Weapon	Range	S.D.	n.
Allied tank	750	674	65
German tank	1290	702	27
Difference	540		

$$\text{p.e.} = \frac{2}{3}\sqrt{\frac{674^2}{65} + \frac{702^2}{27}} = 106$$

Significance test: $\frac{\text{Difference}}{\text{p.e.}} = \frac{540}{106} = 5.1$ ∴ Significant

4. Success ranges of Allied 17 pdr. tank against German S.P. gun and against German towed gun

Opponent	Range	S.D.	n.
German S.P. gun	1270	1512	10
German towed gun	455	89	11
Difference	815		

$$\text{p.e.} = \frac{2}{3}\sqrt{\frac{1512^2}{10} + \frac{89^2}{11}} = 319$$

Significance test: $\frac{\text{Difference}}{\text{p.e.}} = \frac{815}{319} = 2.5$ ∴ NOT Significant

5. Success ranges of Allied S.P. gun and of Allied towed gun against German tanks (all types)

Weapon	Range	S.D.	n.
Allied S.P. gun	1090	715	34
Allied towed gun	870	414	30
Difference	220		

$$\text{p.e.} = \frac{2}{3}\sqrt{\frac{715^2}{34} + \frac{414^2}{30}} = 96$$

Significance test: $\frac{\text{Difference}}{\text{p.e.}} = \frac{220}{96} = 2.3$. ∴ NOT Significant

6. Success ranges of German S.P. gun and of German towed gun against Allied tanks (all types)

Weapon	Range	S.D.	n.
German S.P. gun	330	195	7
German towed gun	300	214	4
Difference	30		

$$\text{p.e.} = \frac{2}{3}\sqrt{\frac{195^2}{7} + \frac{214^2}{4}} = 62$$

Significance test: $\frac{\text{Difference}}{\text{p.e.}} = \frac{30}{62} = 0.5$ ∴ NOT Significant

7. Success ranges of German S.P. gun and of Allied S.P. gun against tanks

Weapon	Range	S.D.	n.
German S.P. gun	330	195	7
Allied S.P. gun	1090	715	34
Difference	760		

$$\text{p.e.} = \frac{2}{3}\sqrt{\frac{195^2}{7} + \frac{715^2}{34}} = 95$$

Significance test: $\frac{\text{Difference}}{\text{p.e.}} = \frac{760}{95} = 8$ ∴ Significant

8. Success ranges of Allied towed A/Tk. gun and of German towed A/Tk. gun against tanks

Weapon	Range	S.D.	n.
Allied towed A/Tk. gun	870	414	30
German towed A/Tk. gun	300	214	14
Difference	570		

$$\text{p.e.} = \frac{2}{3}\sqrt{\frac{414^2}{30} + \frac{214^2}{14}} = 95$$

Significance test: $\frac{\text{Difference}}{\text{p.e.}} = \frac{570}{95} = 6$ ∴ Significant

Appendix F - I

ATTACKERS AND DEFENDERS LOSSES

TANK v. TANK

1. ALLIED attackers in successful attacks.

(a) No. of actions	(b) No. of attackers in each action.	(c) Total No. attackers.	(d) No. of losses in each action.	(e) Total No. of losses	(f) % Losses
9	1	10	0	0	-
1	1	1	1	1	100
3	2	6	0	0	-
1	2	2	1	1	50
3	3	9	0	0	-
1	3	3	1	1	33
3	4	12	0	0	-
1	5	5	0	0	-
1	5	5	1	1	20
1	5	5	2	2	40
24		58		6	10

2. ALLIED attackers in unsuccessful attacks.

(a)	(b)	(c)	(d)	(e)	(f)
4	1	4	1	4	100
1	2	2	0	0	-
3	3	9	2	6	66
1	7	7	4	4	55
9		22		14	64

3. ENEMY attackers in unsuccessful attacks.

(a)	(b)	(c)	(d)	(e)	(f)
6	1	6	1	6	100
1	2	2	2	2	100
2	4	8	2	4	50
1	5	5	3	3	60
1	6	6	3	3	50
11		27		18	66

There are only 4 successful Enemy tank attacks, and in these they suffered 20% losses.

DEFENDERS LOSSES.

TANK v. ANTI-TANK GUN.

4. ALLIED SP gun defenders, successful.

(a)	(b)	(c)	(d)	(e)	(f)
2	1	2	0	0	-
4	2	8	0	0	-
2	3	6	0	0	-
8	4	32	0	0	-
1	4	4	1	1	25
1	4	4	2	2	50
1	4	4	3	3	75
2	8	16	0	0	-
1	12	12	3	3	25
1	12	12	6	6	50
1	24	24	9	9	38
1	36	36	6	6	16
25		160		30	12

DEFENDERS LOSSES

TANK v. ANTI-TANK GUN.

5. ALLIED SP gun defenders, unsuccessful.

(a) No. of actions	(b) No. of attackers in each action.	(c) Total No. attackers.	(d) No. of losses in each action	(e) Total No. of losses	(f) % Losses
1	4	4	0	0	-
1	4	4	4	4	100
1	2	2	2	2	100
1	12	12	6	6	50
4		22		12	54

6. ENEMY SP gun defenders, successful.

(a) No. of actions	(b) No. of defenders in each action	(c) Total No. of defrs.	(d) No. of losses in each action	(e) Total No. of losses	(f) % Losses
7	1	7	0	0	-
1	2	2	0	0	-
1	2	2	1	1	50
2	3	6	0	0	-
1	5	5	2	2	40
12		22		3	13

Where the Enemy defence was unsuccessful, the losses were 100%.

7. ALLIED towed gun defenders, successful.

(a)	(b)	(c)	(d)	(e)	(f)
1	1	1	0	0	-
1	2	2	0	0	-
1	2	2	1	1	50
1	3	3	2	2	66
1	4	4	0	0	-
2	4	8	1	2	25
2	4	4	2	4	50
1	8	8	0	0	-
1	12	12	0	0	-
11		44		9	20

8. ALLIED towed gun defenders, unsuccessful.

(a)	(b)	(c)	(d)	(c)	(f)
1	1	1	1	1	100
2	2	4	1	2	50
4	2	8	2	8	100
1	4	4	2	2	50
1	4	4	3	3	75
2	4	8	4	8	100
1	6	6	6	6	100
1	8	8	8	8	100
1	12	12	8	8	66
14		55		46	84

9. ENEMY towed gun defenders, successful.

(a)	(b)	(c)	(d)	(e)	(f)
7	1	7	0	0	-
1	2	2	0	0	-
1	2	2	1	1	50
9		11		1	9

Enemy towed gun losses were 100% when the defence was not successful.

Distribution List:-

S.W/AC	(2)
D.M.T.	(I)
D.C.I.G.S.	(I)
D.W.D.	(I)
D.R.A.C.	(I)
D.R.A.	(I)
War Office Library	(2)
Director of Naval Operational Research	(I)
D.D.Science, Air Ministry	(I)
D.G.S.R.(D)	(3)
D.G. of A.	(I)
D.G.F.V.	(I)
C.M.H.Q.	(6)
Australian Army Staff	(4)
N.Z.M.H.Q.	(2)
Military Adviser Union of South Africa.	(4)
War Office, India Office	(4)